

VISCOUS ELECTRON FLOW IN GRAPHENE HETEROSTRUCTURES

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Abstract

The article deals with hydrodynamic electron transport in clean two-dimensional systems, where electron-electron interactions dominate and allow the phenomenon of superballistic conduction to be observed. This regime is characterised by an electrical resistance lower than the ballistic limit, thanks to the collective flow of electrons. In the experiments, superlattices of antidots were used to study how the geometry affects the collective transport, showing that certain configurations allow minimising non-hydrodynamic scattering. Using simulations based on the Boltzmann equation and advanced experimental techniques, phenomena such as the Gurzhi effect, weak localisation and the relationship between geometrical structures and the cyclotron radius in the presence of magnetic fields were analysed. The results highlight the importance of electron-electron interactions and geometry in enhancing transport efficiency. These findings may be key to designing low-power devices and more efficient technologies, opening up new possibilities in the study of materials and two-dimensional systems.

INTRODUCTION: BACKGROUND AND MOTIVATION

Recent advancements in materials science have opened the possibility of observing conditions where electrons in metals behave as highly viscous fluids, displaying intriguing hydrodynamic properties. These behaviours, however, have remained difficult to appreciate, primarily due to the challenge of finding materials with the required characteristics. In theory, the "intrinsic" hydrodynamic behaviour of electrons can be recovered if the material is ultraclean (to minimize impurity scattering) and cooled to sufficiently low temperatures to suppress phonon scattering. In 1963, Soviet theorist Radii Gurzhi proposed that in a metallic system, if the electron-electron mean free path (l_{ee}) is the shortest length scale in the system, frequent collisions between electrons would induce a collective flow. In such scenario, the total momentum of the electron system would not be dissipated, allowing for the emergence of viscous electronic flow. In this regime, highly correlated electrons flow collectively, analogous to molecules in viscous fluids [2]. This phenomenon, known as superballistic conduction, is characterised by an electrical resistance below the ballistic limit, making it promise for the development of low-power devices. In contrast, if impurities or thermal vibrations dominate, they would destroy this collective behaviour and prevent the electrons

from exhibiting viscous flow. Gurzhi predicted that the resistivity (R) of such a metal would decrease with increasing temperature, in opposition to the standard behaviour observed in conventional metals, where resistivity increases with temperature—a phenomenon now known as the Gurzhi effect. However, realising these conditions in practice proved to be nearly impossible in most conventional metals. In typical three-dimensional (3D) metals, the mean free path of electron-electron scattering has a weaker inverse temperature dependence than other scattering processes, such as electron-phonon interactions, due to constraints imposed by the Pauli exclusion principle and Fermi-Dirac statistics. At higher temperatures, electron-phonon scattering becomes increasingly relevant, further complicating the observation of viscous electron behaviour. Fortunately, the development of new high-quality materials, particularly graphene, has transformed this scenario. The encapsulation of graphene in hexagonal boron nitride (hBN) significantly improved its electronic quality by shielding it from impurities and creating a flat, stiff surface that suppresses phonon scattering and increases the electron-phonon mean free path. Furthermore, in contrast to 3D metals, electron-phonon scattering in two-dimensional (2D) graphene increases more slowly with temperature, while electron-electron scattering grows more rapidly, making graphene an ideal platform for studying viscous electron flows. Graphene, with its unique electronic properties and high material quality, presents an excellent candidate for the exploration of viscous electron behaviour,

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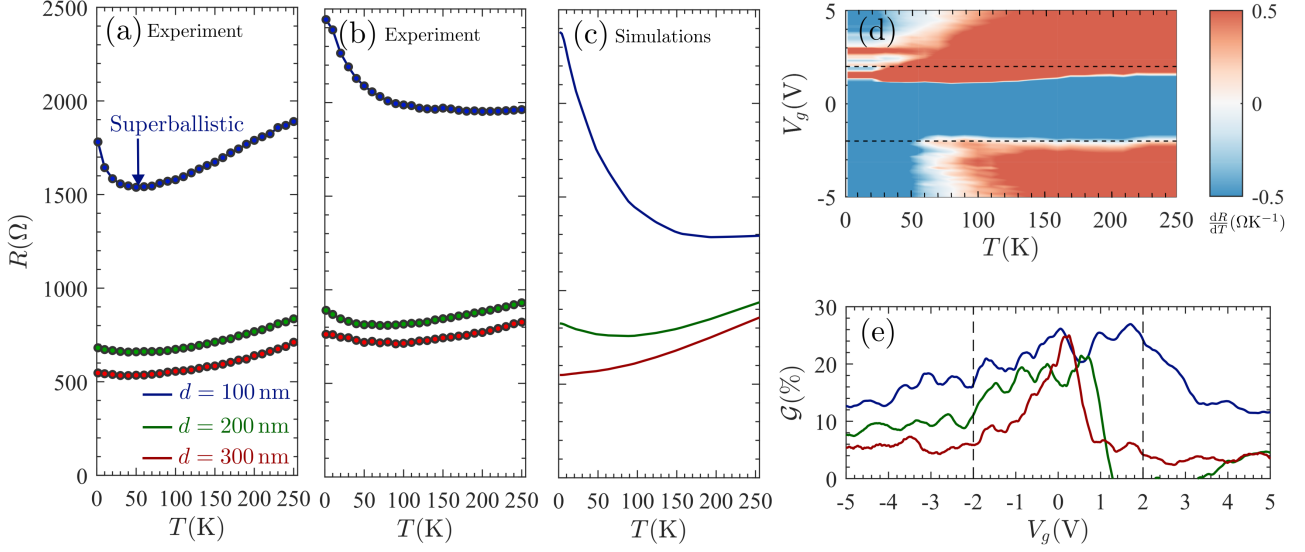


Figure 1: Superballistic electrons. Experimental resistance in the three regions of the antidot lattice with $d = 100$, 200 , and 300 nm as a function of temperature for (a) electrons ($V_g = 3.9$ V) and (b) holes ($V_g = -3.9$ V), V_g defined respect to the Dirac point, and (c) theoretical resistance calculated by Boltzmann equation simulations. (e) Gurzhi ratio $\mathcal{G}$ as a function of V_g for the three antidot regions.[1]

offering new opportunities to study hydrodynamic electron flows in solid-state systems.

SUPERBALLISTIC ELECTRONS

The superballistic regime is given when the electrical resistance in a system decreases below the ballistic limit due to a hydrodynamic flow of electrons. In contrast to conventional regimes dominated by collisions with impurities or phonons, superballistic transport is characterised by higher efficiency, with potential implications for low-power electronic device technology. This phenomenon is studied in hBN-encapsulated graphene heterostructures with superlattice geometries of antidots by observing a ballistic-hydrodynamic transition. These geometries, aided by the techniques used for the fabrication of these devices, amplify and enhance the hydrodynamic effects.

For the fabrication of the superlattice of antidots in encapsulated graphene heterostructures, the standard mechanical exfoliation process was started. This process allows us to isolate and separate monoatomic layers of graphene and hexagonal boron nitride obtained through pristine crystals. Thanks to this process we will obtain the basic components of the heterostructure. A Bruker DekTakXT profilometer (Fig. 2(b)) was used to ensure the necessary requirements for the fabrication of the samples [1]. This equipment accurately measures the thickness of the layers to guarantee the quality of the samples. Once the basic heterostructure was

fabricated, an electron beam lithography (EBL) (Fig. 2(a)) system was used to shape the device into a Hall bar with ten terminals. This step ensures the precise geometry needed to study the transport phenomena. Subsequently, cryogenic etching was carried out with an ICP-RIE system (PlasmaPro 100 Estrelas - Fig. 2(c) and Fig. 2(d)), which produces soft edges essential to minimise unwanted effects such as inelastic electron scattering and to ensure smooth edges for obtaining electron beam reflection [3]. To define the periodic geometric antidot patterns, the EBL system was again used in conjunction with cryogenic etching (Fig. 2(c) and Fig. 2(d)). Finally, to electrically connect the device, metal contacts were deposited with an electron beam evaporation system (Fig. 3).

Devices were fabricated with three regions of square superlattices of antidots of specific sizes ($d = 100$ nm, 200 nm and 300 nm), with a centre-to-centre distance between antidots equivalent to $2d$ [1]. This configuration allows the study of the influence of antidot dimensions on the collective electron transport.

In turn, these experimental data were complemented with simulations following the Boltzmann semi-classical transport kinetic model to cover both ballistic and superballistic regimes.

The Figure (1) represents the variation of the longitudinal resistance R_{xx} as a function of temperature for the three different antidots regions showing a consistent decrease in resistance as the temperature increases from 1.5 to 50 K. The region with $d = 100$ nm stands out with the largest drop.



Figure 2: USAL-Nanolab Clean Room: a) FE-SEM and Raith nanolithography system. b) Profilometer system. c) Software control and load-lock chamber for PlasmaPro 100 Estrelas ICP-RIE. d) Main room PlasmaPro 100 Estrelas ICP-RIE.



Figure 3: USAL-Nanolab Clean Room: E-beam evaporator.

The photos were taken in the Clean Room of the Nanotechnology Group of Salamanca university.

In section (c) of this same Figure (1) we can see the simulations of the theoretical resistance calculated from the Boltzmann equation. The reduction of resistance in the superballistic regime

is highlighted, showing that at low temperatures the longitudinal resistance (R_{xx}) in the three antidot regions is significantly reduced below the ballistic limit, which confirms the presence of superballistic transport. At the same time, the region with the smallest diameter antidots ($d = 100$ nm) exhibited the greatest reduction in resistance, reaching up to 20% less than the ballistic limit at $T = 50$ K. On the other hand, regions with $d = 200$ nm and $d = 300$ nm showed more moderate effects. Simulations of the Boltzmann equation reproduced the experimental data with high reliability, validating the importance of electron-electron interactions and the geometry of the antidots. The region with $d = 100$ nm, due to its more favourable ratio between antidot diameter (d) and electron mean free path (l_{ee}), maximises the inhomogeneity of the flow, resulting in higher collectivisation and lower resistance. The heat map (Fig. 1(d)) illustrates the derivative of resistance with respect to temperature, highlighting areas where superballistic conduction is dominant. The blue zone of the colormap therefore corresponds to a negative slope, characteristic of superballistic conduction, representing optimal electronic collectivisation conditions. After a certain temperature, interaction with phonons and impurities becomes more relevant, reflected in the resistance increase (red area of the colormap).

Finally, a quantity called the Gurzhi ratio $\mathcal{G} = 1 - R(50\text{ K})/R(1.5\text{ K})$ which offers us greater visibility of the superballistic effect by providing us with information to distinguish the real superballistic effect from other possible physical effects occurring at higher temperatures, shown in Figure 1(e).

The results obtained confirm that superlattices of antidots are an ideal platform for the study and investigation of the hydrodynamic regime. The geometry of the antidots plays an important role in the trajectory of the electrons, leaving aside its relation with the other characteristic quantities to observe the Gurzhi effect. This geometrical design amplifies the collectivisation of the flow and minimises non-hydrodynamic scattering, which allows the collective transport to dominate even at higher temperatures. The implications of these findings are significant. On the one hand, they demonstrate the potential of hydrodynamic transport-based devices for low-power technology applications. On the other hand, they open up new questions on how to further optimise these systems through 2D material combinations and more complex geometric designs.

MAGNETOHYDRODYNAMIC

Graphene, with its unique electronic properties and high mobility, offers an ideal environment to study magnetotransport under extreme conditions. We explore how magnetic fields influence electron transport, highlighting key phenomena such as weak localisation, the quantum Hall effect (QHE) and commensurability in the hydrodynamic regime.

In this case, the measurements were carried out using a Teslatron dry cryostat (Fig. 4(a)) as the main component of the experimental setup. This device provides cryogenic cooling by means of a closed system avoiding the use of liquids for cooling. Thanks to the use of efficient heat exchangers, stable temperatures are maintained. At the same time, this device incorporates a superconducting magnet which generates magnetic fields oriented perpendicularly to the Hall device, and therefore this configuration opens the door to the subsequent study of magnetotransport phenomena. Using specialised software, the intensity of the magnetic field and its orientation are controlled to improve experimental precision. Using the Lock-In amplifier devices (Fig. 4(b)), very weak signals are detected, as the unwanted components are filtered out and the excitation signal is modulated to reduce the environmental noise. Both the AC current applied to the device is monitored and the voltage recorded with high accuracy. On the other hand, the sourcemeters (Fig. 4(b)) are responsible for modulating the charge carrier density in the material [3].

The photos were taken in the Low Temperature Room of the Faculty of Physics of the University of Salamanca.

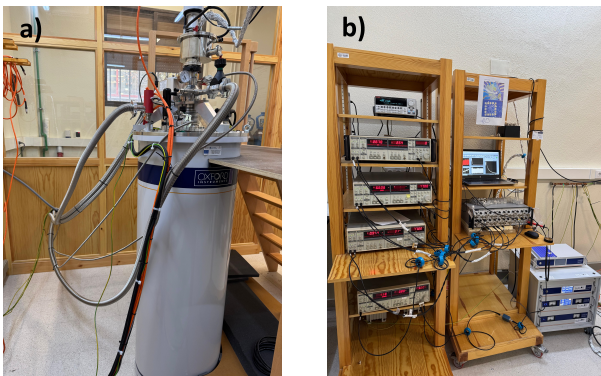


Figure 4: USAL-Nanolab Low Temperature Laboratory. *a)* Variable Temperature Insert for a Teslatron dry cryostat with a 12 T magnet. *b)* Measurement rack with several lock-in amplifiers and source-meters.

Magnetotransport describes the response of electrons to magnetic fields applied perpendicularly to a material. Under a magnetic field, electrons

follow curved orbits with cyclotron radio (l_b) and are affected by the geometrical properties of the device.

Weak localisation is a quantum phenomenon in which scattered electrons experience constructive interference, increasing electrical resistance when scattered in closed paths at low magnetic fields ($B \leq 20$ mT). This phenomenon is most noticeable in clean systems such as hBN-encapsulated graphene, where collisions are dominated by well-defined edges and antidots, and scattering by phonons and impurities is minimised. Backscattering trajectories (when an electron returns along the same path) generate constructive interference between electron waves travelling in opposite directions, thereby increasing the electrical resistance as the probability of backscattering this interference increases.

In devices with simple geometries, such as Hall bars, this phenomenon occurs in the presence of homogeneous regions of carrier density.

Under normal conditions, electrons move freely without impositions and their trajectory is random. But if subjected to a high magnetic field perpendicular to the plane of the electronic system in two dimensions (2DEG), the electrons will be subject to the Lorentz force that bends their trajectory making them rotate in circular orbits around the magnetic field lines, discretising their energy into quantized levels known as Landau levels (LLs) if the magnetic field is enough high. At low temperatures, the electrons occupy these levels in an ordered fashion, quantizing the conductivity into discrete levels of a fundamental constant called the von Klitzing constant (h/e^2). In turn, quantization of the Hall resistance occurs. This is known as the Quantum Hall Effect (QHE).

$$\sigma_{xy} = \frac{e^2}{h} \nu = \frac{\nu}{R_k}$$

where $\nu = \frac{nh}{eB}$ is the filling factor and can be either integer (integer quantum Hall effect IQHE) or fractional (fractional quantum Hall effect FQHE) [3]. For monolayer graphene, the Landau levels are four times degenerate, two for the spin and two for the valley.

The commensurability effect (CE) gives rise to a hump when the cyclotron radius (l_b) is comparable to the size of the antidot lattice, which means that electrons from the edge of one hole stream into another hole, which logically hinders electron transport. This effect can also be observed when l_b is proportional to the width of the Hall bar. In the hydrodynamic regime, this coincidence amplifies the collective transport, as the electron flows adapt to the geometry, reducing the resistance.

Figure (5) shows the results obtained. In the superlattices of antidots a peak in longitudinal

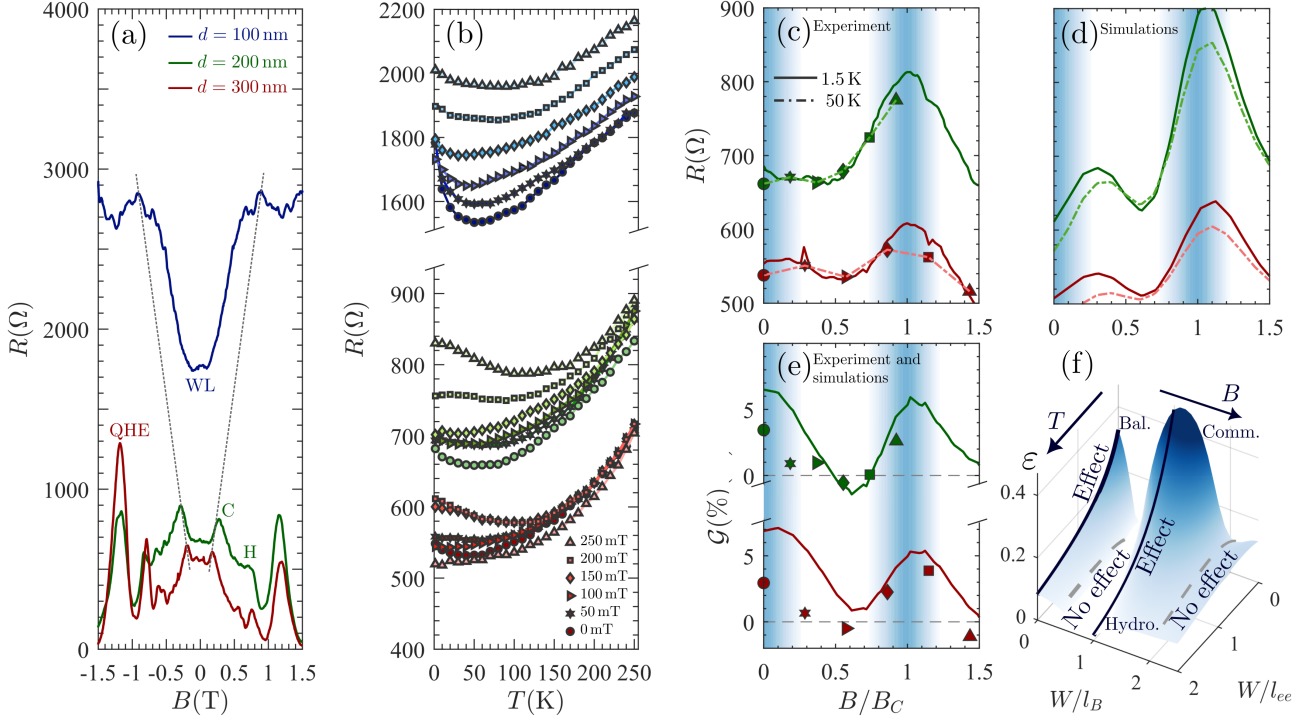


Figure 5: Magnetohydrodynamic. (a) Longitudinal resistance for the three regions at $V_g = 3.9$ V and $T = 1.5$ K shows weak localization (WL), commensurability effect (C), the hydrodynamic region (H), and oscillations associated with the quantum Hall effect (QHE). (b) Superballistic effect for several magnetic fields. (c) Experimental measurements and (d) Boltzmann equation simulations of the resistance as a function of the magnetic field, normalized to the commensurability fields B_C of the peak. (e) Gurzhi ratio $\mathcal{G}$, symbols are experimental observations and the lines are simulated values. (f) Qualitative explanation of the intermittent effect, where W is the typical width of a uniform device.[1]

resistance was observed due to weak localization, since quantum interference is intensified in smaller geometries as electrons interact more frequently with the curved edges of the antidots. Most relevant is the intermittency of the hydrodynamic regime itself. When the magnetic field reaches the vicinity of the magnetic field $B_C \approx 1.05 \hbar \sqrt{\pi n} / ed$, which is related to the fact that the cyclotron radius is commensurate with the size of the antidot lattice [1]. This commensurability region can be better visualized experimentally in Fig. 5(c) and simulated in Fig. 5(d). Fig. 5(b) shows the dependence of the resistance on temperature for different fixed values of the magnetic field. It can be seen how, depending on its value, the hydrodynamic regime is shown for some cases and not for others. As in the case of the superballistic analysis, we will use Gurzhi's parameter to better discern when this effect occurs. If the Gurzhi parameter is represented as a function of the generic magnetic field B and particular commensurability B_C , it can be seen that there are not only hydrodynamic phenomena in the absence of magnetic field but also the hydrodynamic regime and superballistic transport when the magnetic field meets these conditions of commensurability mentioned. Looking ahead,

research may focus on further optimizing antidot lattice geometries and exploring combinations with other two-dimensional materials to improve thermal stability and extend the operating range of the hydrodynamic regime. Incorporation of advanced fabrication techniques, such as controlled stacking and electron beam lithography, could enable more complex structures that maximize flow collectivization. It is also promising to study the intermittency of the ballistic and hydrodynamic regimes by analyzing transitions under different temperature and magnetic field conditions. In particular, analysis of the commensurability between the diameter of antidots and the cyclotron radius will be key to advancing the design of devices that take full advantage of the collective properties of graphene.

CONCLUSIONS AND OUTLOOK

The study of hydrodynamic flow in encapsulated graphene heterostructures confirms the potential of this material to advance the understanding of electronic transport regimes and the convenience of the use of antidot graphene superlattices.

Through experimental designs such as superlattices of antidots and Hall bar devices, phenomena such as superballistic conduction, quantum Hall effect and commensurability in the hydrodynamic regime have been explored. These investigations highlight how geometrical features amplify the collectivization of electronic flow, enabling more efficient transport regimes to be achieved than traditional ballistic ones. In addition, theoretical tools, such as Boltzmann equation simulations, support the experimental findings and underline the importance of electron-electron interactions in the hydrodynamic regime. This work also lays the groundwork for technological applications, such as materials to reduce the resistance in technological devices, quantum devices and energy-efficient components.

During these months I learned the basic steps for fabricate such a heterostructures and I hope to be aim to continue this work during the next moths including a region without holes and a matrix with antidots of 50 nm of diameter.

I would like to thank Juan Salvador-Sánchez, Vito Clérico, Ana Pérez-Rodríguez and Mario Amado, as well as my tutor Enrique Diez, for their operational guidance on the artisanal fabrication of 2D material heterostructures and their characterization.

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